Solving Equations and Systems of Equations

MODULE 7

Solving Linear Equations

FL 8.EE.3.7, 8.EE.3.7a, 8.EE.3.7b

MODULE 8

Solving Systems of Linear Equations

FL 8.EE.3.8, 8.EE.3.8a, 8.EE.3.8b, 8.EE.3.8c

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CAREERS IN MATH

Hydraulic Engineer A hydraulic engineer specializes in the behavior of fluids, mainly water. A hydraulic engineer applies the mathematics of fluid dynamics to the collection, transport, measurement, and regulation of water and other fluids.

If you are interested in a career in hydraulic engineering, you should study the following mathematical subjects:
- Algebra
- Geometry
- Trigonometry
- Probability and Statistics
- Calculus

Research other careers that require the understanding of the mathematics of fluid dynamics.

Unit 3 Performance Task

At the end of the unit, check out how **hydraulic engineers** use math.

Vocabulary Preview

Use the puzzle to preview key vocabulary from this unit. Unscramble the circled letters to answer the riddle at the bottom of the page.

1. **FIACALRONT INFEOCIECTF**

2. **LCMADEI CINETFOEFIC**

3. **UQAOTENI**

4. **ROPWE**

5. **TNUSITBUOSIT DOHMTE**

1. A number that is multiplied by the variable in an algebraic expression, where the number is a fraction. **(Lesson 7.2)**

2. A number that is multiplied by the variable in an algebraic expression, where the number is a decimal. **(Lesson 7.2)**

3. A mathematical statement that two expressions are equal. **(Lesson 7.1)**

4. A number that is formed by repeated multiplication of the same factor. Multiply by this to remove decimals from an unsolved equation. **(Lesson 7.2)**

5. A process used to solve systems of linear equations by solving an equation for one variable and then substituting the resulting expression for that variable into the other equation. **(Lesson 8.2)**

Q: What is the best time to divide a half dollar between two people?

A: at a ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___!

Solving Linear Equations

? ESSENTIAL QUESTION

How can you use equations with the variable on both sides to solve real-world problems?

Real-World Video

Some employees earn commission plus their salary when they make a sale. There may be options about their pay structure. They can find the best option by solving an equation with the variable on both sides.

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Animated Math
Interactively explore key concepts to see how math works.

Personal Math Trainer
Get immediate feedback and help as you work through practice sets.

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Are YOU Ready?

Complete these exercises to review skills you will need for this module.

Find Common Denominators

EXAMPLE Find the LCD of 3, 5, and 10.

3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30,…   List the multiples of each number.
5: 5, 10, 15, 20, 25, 30, 35,…   Choose the least multiple the
10: 10, 20, 30, 40, 50,…   lists have in common.
LCD(3, 5, 10) = 30

Find the LCD.

1. 8, 12 _____   **2.** 9, 12 _____   **3.** 15, 20 _____   **4.** 8, 10 _____

Multiply Decimals by Powers of 10

EXAMPLE 3.719×100 Count the zeros in 100: 2 zeros
$3.719 \times 100 = 371.9$ Move the decimal point 2 places to the right.

Find the product.

5. 0.683×100 **6.** $9.15 \times 1,000$ **7.** 0.005×100 **8.** $1,000 \times 1,000$

_____   _____   _____   _____

Connect Words and Equations

EXAMPLE   Two times a number decreased by 5 is -6.

Two times x decreased by 5 is -6.   Represent the unknown with a variable.
$2x - 5$ is -6 Times means multiplication.
$2x - 5 = -6$ Decreased by means subtraction.
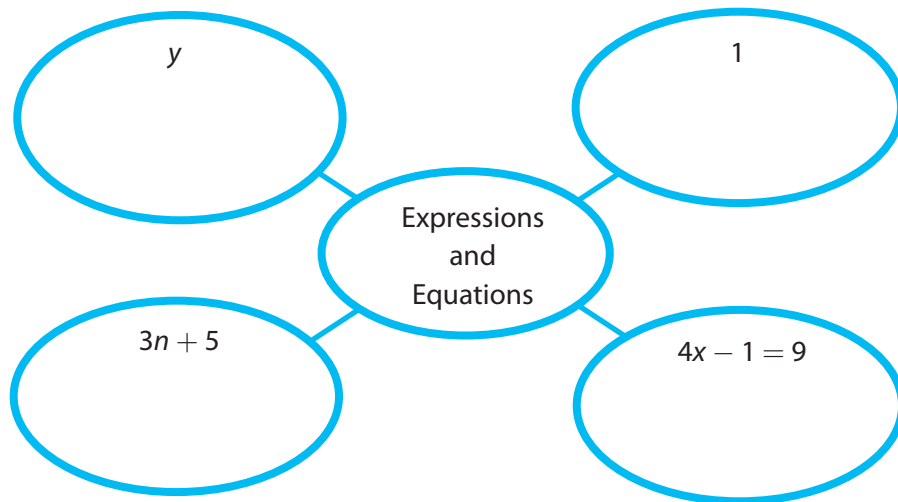Place the equal sign.

Write an algebraic equation for the sentence.

9. The difference between three times a number and 7 is 14. _____

10. The quotient of five times a number and 7 is no more than 10. _____

11. 14 less than 3 times a number is 5 more than half of the number.   _____

Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the bubble map. You may put more than one word in each oval.

```
   ( y )                    ( 1 )
          \              /
        ( Expressions
            and
          Equations )
          /              \
   ( 3n + 5 )            ( 4x - 1 = 9 )
```

Understand Vocabulary

Complete the sentences using the review words.

1. A value of the variable that makes an equation true is a _____.

2. The set of all whole numbers and their opposites are _____.

3. An _____ is an expression that contains at

    least one variable.

Active Reading

Layered Book Before beginning the module, create a layered book to help you learn the concepts in this module. At the top of the first flap, write the title of the book, "Solving Linear Equations." Then label each flap with one of the lesson titles in this module. As you study each lesson, write important ideas, such as vocabulary and formulas, under the appropriate flap.

Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

FL 8.EE.3.7a

Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

Key Vocabulary

linear equation in one variable
(ecuación lineal en una variable)
An equation that can be written in the form $ax = b$ where a and b are constants and $a \neq 0$.

What It Means to You

You will identify the number of solutions an equation has.

UNPACKING EXAMPLE 8.EE.3.7a

Your gym charges $50 per month. Find the number of months for which your costs will equal the cost of membership at each gym shown.

A: $40 per month plus $100 one-time fee
$50x = 40x + 100 \rightarrow x = 10$
Equal in 10 months → one solution

B: $50 per month plus $25 one-time fee
$50x = 50x + 25 \rightarrow 0 = 25$
Never equal → no solution

C: $40 per month plus $10 monthly garage fee
$50x = 40x + 10x \rightarrow x = x$
Equal for any number of months → infinitely many solutions

FL 8.EE.3.7b

Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Key Vocabulary

solution *(solución)*
In an equation, the value for the variable that makes the equation true.

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What It Means to You

You can write and solve an equation that has a variable on both sides of the equal sign.

UNPACKING EXAMPLE 8.EE.3.7b

Yellow Taxi has no pickup fee but charges $0.25 per mile. AAA Taxi charges $3 for pickup and $0.15 per mile. Find the number of miles for which the cost of the two taxis is the same.

$$0.25x = 3 + 0.15x$$
$$100(0.25x) = 100(3) + 100(0.15x)$$
$$25x = 300 + 15x$$
$$10x = 300$$
$$x = 30$$

The cost is the same for 30 miles.

Equations with the Variable on Both Sides

ESSENTIAL QUESTION

How can you represent and solve equations with the variable on both sides?

Modeling an Equation with a Variable on Both Sides

Algebra tiles can model equations with a variable on both sides.

KEY

$+ = 1$

$- = -1$ $| = x$

$+ \ + \ - = 0$

Use algebra tiles to model and solve $x + 5 = 3x - 1$.
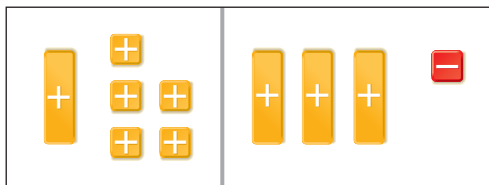
Model $x + 5$ on the left side of the mat and $3x - 1$ on the right side.
Remember that $3x - 1$ is the same as

$3x +$ _____.
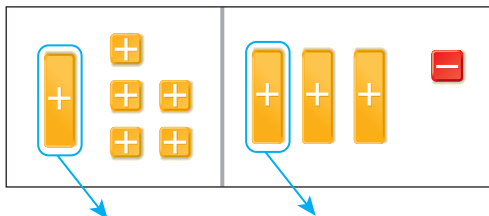
Remove one x-tile from both sides. This

represents subtracting _____
from both sides of the equation.

Math Talk
Mathematical Practices

Why is a positive unit tile added to both sides in the third step?
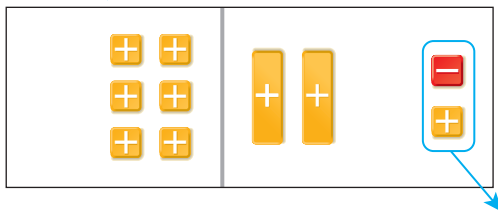
Place one $+1$-tile on both sides. This

represents adding _____ to both sides of the equation. Remove zero pairs.
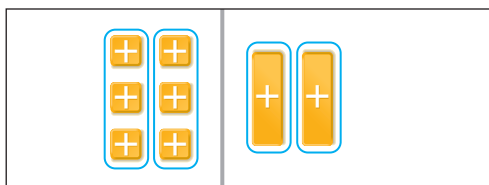
Separate each side into 2 equal groups.

One x-tile is equivalent to _____ $+1$-tiles.
The solution is _____ = _____.

Reflect

1. How can you check the solution to $x + 5 = 3x - 1$ using algebra tiles?

_____

_____

Solving an Equation with the Variable on Both Sides

Equations with the variable on both sides can be used to compare costs of real-world situations. To solve these equations, use inverse operations to get the variable terms on one side of the equation.

**EXAMPLE 1** Real World   FL   8.EE.3.7, 8.EE.3.7b

Andy's Rental Car charges an initial fee of $20 plus an additional $30 per day to rent a car. Buddy's Rental Car charges an initial fee of $36 plus an additional $28 per day. For what number of days is the total cost charged by the companies the same?

STEP 1   Write an expression representing the total cost of renting a car from Andy's Rental Car.

$$\text{Initial fee} \quad + \quad \text{Cost for } x \text{ days}$$
$$20 \quad + \quad 30x$$

STEP 2   Write an expression representing the total cost of renting a car from Buddy's Rental Car.

$$\text{Initial fee} \quad + \quad \text{Cost for } x \text{ days}$$
$$36 \quad + \quad 28x$$

STEP 3   Write an equation that can be solved to find the number of days for which the total cost charged by the companies would be the same.

$$\text{Total cost at Andy's} \quad = \quad \text{Total cost at Buddy's}$$
$$20 + 30x \quad = \quad 36 + 28x$$

STEP 4   Solve the equation for x.

$$
\begin{array}{rcll}
20 + 30x &=& 36 + 28x & \textit{Write the equation.} \\
\underline{-28x} & & \underline{-28x} & \textit{Subtract } 28x \textit{ from both sides.} \\
20 + 2x &=& 36 & \\
\underline{-20} & & \underline{-20} & \textit{Subtract 20 from both sides.} \\
2x &=& 16 & \\
\dfrac{2x}{2} &=& \dfrac{16}{2} & \textit{Divide both sides by 2.} \\
x &=& 8 &
\end{array}
$$

The total cost is the same if the rental is for 8 days.

Math Talk
Mathematical Practices

When is it more economical to rent from Andy's Rental Car? When is it more economical to rent from Buddy's?

2. A water tank holds 256 gallons but is leaking at a rate of 3 gallons per week. A second water tank holds 384 gallons but is leaking at a rate of 5 gallons per week. After how many weeks will the amount of water in the two tanks be the same?

_____

Writing a Real-World Situation from an Equation

As shown in Example 1, an equation with the variable on both sides can be used to represent a real-world situation. You can reverse this process by writing a real-world situation for a given equation.

EXAMPLE 2 🌐 Real World   FL   8.EE.3.7

My Notes

Write a real-world situation that could be modeled by the equation $150 + 25x = 55x$.

STEP 1 The left side of the equation consists of a constant plus a variable term. It could represent the total cost for doing a job where there is an initial fee plus an hourly charge.

STEP 2 The right side of the equation consists of a variable term. It could represent the cost for doing the same job based on an hourly charge only.

STEP 3 The equation $150 + 25x = 55x$ could be represented by this situation: A handyman charges $150 plus $25 per hour for house painting. A painter charges $55 per hour. How many hours would a job have to take for the handyman's fee and the painter's fee to be the same?

3. Write a real-world situation that could be modeled by the equation $30x = 48 + 22x$.

_____

_____

_____

_____

Use algebra tiles to model and solve each equation. (Explore Activity)

1. $x + 4 = -x - 4$ _____

2. $2 - 3x = -x - 8$ _____

3. At Silver Gym, membership is $25 per month, and personal training sessions are $30 each. At Fit Factor, membership is $65 per month, and personal training sessions are $20 each. In one month, how many personal training sessions would Sarah have to buy to make the total cost at the two gyms equal? (Example 1)

_____

4. Write a real-world situation that could be modeled by the equation $120 + 25x = 45x$. (Example 2)

_____

_____

_____

5. Write a real-world situation that could be modeled by the equation $100 - 6x = 160 - 10x$. (Example 2)

_____

_____

_____

_____

_____

? ESSENTIAL QUESTION CHECK-IN

6. How can you solve an equation with the variable on both sides?

_____

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_____

7.1 Independent Practice

FL 8.EE.3.7, 8.EE.3.7b

7. Derrick's Dog Sitting and Darlene's Dog Sitting are competing for new business. The companies ran the ads shown.

a. Write and solve an equation to find the number of hours for which the total cost will be the same for the two services.

_____

b. **Analyze Relationships** Which dog sitting service is more economical to use if you need 5 hours of service? Explain.

_____

_____

Derrick's Dog Sitting

$12 plus $5 per hour

Darlene's Dog Sitting

$18 plus $3 per hour

8. Country Carpets charges $22 per square yard for carpeting, and an additional installation fee of $100. City Carpets charges $25 per square yard for the same carpeting, and an additional installation fee of $70.

a. Write and solve an equation to find the number of square yards of carpeting for which the total cost charged by the two companies will be the same.

_____

b. **Justify Reasoning** Mr. Shu wants to hire one of the two carpet companies to install carpeting in his basement. Is he more likely to hire Country Carpets or City Carpets? Explain your reasoning.

_____

_____

_____

_____

Write an equation to represent each relationship. Then solve the equation.

9. Two less than 3 times a number is the same as the number plus 10.

_____

10. A number increased by 4 is the same as 19 minus 2 times the number.

_____

11. Twenty less than 8 times a number is the same as 15 more than the number.

_____

12. The charges for an international call made using the calling card for two phone companies are shown in the table.

Phone Company	Charges
Company A	35¢ plus 3¢ per minute
Company B	45¢ plus 2¢ per minute

a. What is the length of a phone call that would cost the same no matter which company is used?

_____

b. **Analyze Relationships** When is it better to use the card from Company B?

_____

_____

H.O.T. **FOCUS ON HIGHER ORDER THINKING**

13. **Draw Conclusions** Liam is setting up folding chairs for a meeting. If he arranges the chairs in 9 rows of the same length, he has 3 chairs left over. If he arranges the chairs in 7 rows of that same length, he has 19 left over. How many chairs does Liam have?

_____

14. **Explain the Error** Rent-A-Tent rents party tents for a flat fee of $365 plus $125 a day. Capital Rentals rents party tents for a flat fee of $250 plus $175 a day. Delia wrote the following equation to find the number of days for which the total cost charged by the two companies would be the same:

$$365x + 125 = 250x + 175$$

Find and explain the error in Delia's work. Then write the correct equation.

_____

_____

_____

15. **Persevere in Problem Solving** Lilliana is training for a marathon. She runs the same distance every day for a week. On Monday, Wednesday, and Friday, she runs 3 laps on a running trail and then runs 6 more miles. On Tuesday and Sunday, she runs 5 laps on the trail and then runs 2 more miles. On Saturday, she just runs laps. How many laps does Lilliana run on Saturday?

_____

LESSON 7.2 Equations with Rational Numbers

? ESSENTIAL QUESTION

How can you solve equations with rational number coefficients and constants?

Solving an Equation that Involves Fractions

To solve an equation with the variable on both sides that involves fractions, start by eliminating the fractions from the equation.

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EXAMPLE 1
FL 8.EE.3.7b, 8.EE.3.7

Solve $\frac{7}{10}n + \frac{3}{2} = \frac{3}{5}n + 2$.

$$10 = 10 \times 1$$
$$= 5 \times 2$$
$$= 2 \times 5$$

STEP 1 Determine the least common multiple of the denominators: LCM(**10**, **5**, **2**) = 10

STEP 2 Multiply both sides of the equation by the LCM.

$$10\left(\frac{7}{10}n + \frac{3}{2}\right) = 10\left(\frac{3}{5}n + 2\right)$$

$$\overset{1}{10}\left(\frac{7}{10}n\right) + \overset{5}{10}\left(\frac{3}{2}\right) = \overset{2}{10}\left(\frac{3}{5}n\right) + 10(2)$$

$$7n + 15 = 6n + 20$$

STEP 3 Use inverse operations to solve the equation.

$$
\begin{array}{rcl}
7n + 15 & = & 6n + 20 \\
\underline{-15} & & \underline{-15} \\
7n & = & 6n + 5 \\
\underline{-6n} & & \underline{-6n} \\
n & = & 5
\end{array}
$$

Subtract 15 from both sides.

Subtract 6n from both sides.

Math Talk
Mathematical Practices

The constant on the right side, 2, is not a fraction. Why do you still need to multiply it by the LCM, 10?

Reflect

1. What is the advantage of multiplying both sides of the equation by the least common multiple of the denominators in the first step?

2. **What If?** What happens in the first step if you multiply both sides by a common multiple of the denominators that is not the LCM?

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YOUR TURN

Solve.

3. $\frac{1}{7}k - 6 = \frac{3}{7}k + 4$ _____

4. $\frac{5}{6}y + 1 = -\frac{1}{2}y + \frac{1}{4}$ _____

Solving an Equation that Involves Decimals

Solving an equation with the variable on both sides that involves decimals is similar to solving an equation with fractions. But instead of first multiplying both sides by the LCM, multiply by a power of 10 to eliminate the decimals.

EXAMPLE 2 | Real World | FL | 8.EE.3.7, 8.EE.3.7b

Javier walks from his house to the zoo at a constant rate. After walking 0.75 mile, he meets his brother, Raul, and they continue walking at the same constant rate. When they arrive at the zoo, Javier has walked for 0.5 hour and Raul has walked for 0.2 hour. What is the rate in miles per hour at which the brothers walked to the zoo?

STEP 1 Write an equation for the distance from the brothers' house to the zoo, using the fact that distance equals rate times time. Let $r = $ the brothers' walking rate.

$$\underbrace{\text{distance to zoo}}_{0.2r + 0.75} = \underbrace{\text{distance to zoo}}_{0.5r}$$

STEP 2 Multiply both sides of the equation by $10^2 = 100$.

> Multiplying by 100 clears the equation of decimals. Multiplying by 10 does not: $10 \times 0.75 = 7.5$.

$$100(0.2r) + 100(0.75) = 100(0.5r)$$
$$20r + 75 = 50r$$

STEP 3 Use inverse operations to solve the equation.

$$
\begin{array}{rcl}
20r + 75 & = & 50r \\
-20r & & -20r \\
\hline
75 & = & 30r \\
\dfrac{75}{30} & = & \dfrac{30r}{30} \\
2.5 & = & r
\end{array}
$$

Write the equation.
Subtract 20r from both sides.

Divide both sides by 30.

So, the brothers' constant rate of speed was 2.5 miles per hour.

5. Logan has two aquariums. One aquarium contains 1.3 cubic feet of water and the other contains 1.9 cubic feet of water. The water in the larger aquarium weighs 37.44 pounds more than the water in the smaller aquarium. Write an equation with a variable on both sides to represent the situation. Then find the weight of 1 cubic foot of water.

_____

Writing a Real-World Situation from an Equation

Real-world situations can often be represented by equations involving fractions and decimals. Fractions and decimals can represent quantities such as weight, volume, capacity, time, and temperature. Decimals can also be used to represent dollars and cents.

EXAMPLE 3 🌎 Real World   FL   8.EE.3.7

Write a real-world situation that can be modeled by the equation $0.95x = 0.55x + 60$.

The left side of the equation consists of a variable term. It could represent the total cost for x items.

The right side of the equation consists of a variable term plus a constant. It could represent the total cost for x items plus a flat fee.

The equation $0.95x = 0.55x + 60$ could be represented by this situation: Toony Tunes charges $0.95 for each song you download. Up With Downloads charges $0.55 for each song but also charges an annual membership fee of $60. How many songs must a customer download in a year so that the cost will be the same at both websites?

My Notes

YOUR TURN

6. Write a real-world problem that can be modeled by the equation $\frac{1}{3}x + 10 = \frac{3}{5}x$.

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1. Sandy is upgrading her Internet service. Fast Internet charges $60 for installation and $50.45 per month. Quick Internet has free installation but charges $57.95 per month. **(Example 2)**

    a. Write an equation that can be used to find the number of months at which the Internet service would cost the same.

    _____

    b. Solve the equation.

    _____

Solve. (Examples 1 and 2)

2. $\frac{3}{4}n - 18 = \frac{1}{4}n - 4$

3. $6 + \frac{4}{5}b = \frac{9}{10}b$

4. $\frac{2}{11}m + 16 = 4 + \frac{6}{11}m$

5. $2.25t + 5 = 13.5t + 14$

6. $3.6w = 1.6w + 24$

7. $-0.75p - 2 = 0.25p$

8. Write a real-world problem that can be modeled by the equation $1.25x = 0.75x + 50$. **(Example 3)**

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_____

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_____

? **ESSENTIAL QUESTION CHECK-IN**

9. How does the method for solving equations with fractional or decimal coefficients and constants compare with the method for solving equations with integer coefficients and constants?

_____

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7.2 Independent Practice

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10. Members of the Wide Waters Club pay $105 per summer season, plus $9.50 each time they rent a boat. Nonmembers must pay $14.75 each time they rent a boat. How many times would a member and a non-member have to rent a boat in order to pay the same amount?

_____

11. Margo can purchase tile at a store for $0.79 per tile and rent a tile saw for $24. At another store she can borrow the tile saw for free if she buys tiles there for $1.19 per tile. How many tiles must she buy for the cost to be the same at both stores?

_____

12. The charges for two shuttle services are shown in the table. Find the number of miles for which the cost of both shuttles is the same.

	Pickup Charge ($)	Charge per Mile ($)
Easy Ride	10	0.10
Best	0	0.35

_____

13. **Multistep** Rapid Rental Car charges a $40 rental fee, $15 for gas, and $0.25 per mile driven. For the same car, Capital Cars charges $45 for rental and gas and $0.35 per mile.

a. For how many miles is the rental cost at both companies the same?

_____

b. What is that cost?

_____

14. Write an equation with the solution $x = 20$. The equation should have the variable on both sides, a fractional coefficient on the left side, and a fraction anywhere on the right side.

_____

15. Write an equation with the solution $x = 25$. The equation should have the variable on both sides, a decimal coefficient on the left side, and a decimal anywhere on the right side. One of the decimals should be written in tenths, the other in hundredths.

_____

16. **Geometry** The perimeters of the rectangles shown are equal. What is the perimeter of each rectangle?

$n + 0.1$

$n + 0.6$

n

$2n$

_____
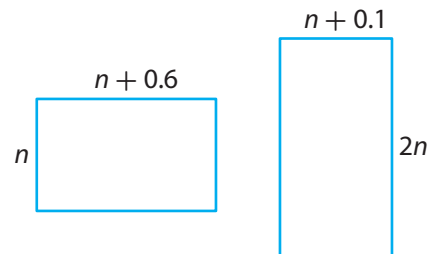
17. **Analyze Relationships** The formula $F = 1.8C + 32$ gives the temperature in degrees Fahrenheit (F) for a given temperature in degrees Celsius (C). There is one temperature for which the number of degrees Fahrenheit is equal to the number of degrees Celsius. Write an equation you can solve to find that temperature and then use it to find the temperature.

_____

18. Explain the Error Agustin solved an equation as shown. What error did Agustin make? What is the correct answer?

$$\frac{1}{3}x - 4 = \frac{3}{4}x + 1$$

$$12\left(\frac{1}{3}x\right) - 4 = 12\left(\frac{3}{4}x\right) + 1$$

$$4x - 4 = 9x + 1$$

$$-5 = 5x$$

$$x = -1$$

_____

_____

_____

_____

H.O.T.   **FOCUS ON HIGHER ORDER THINKING**

19. Draw Conclusions Solve the equation $\frac{1}{2}x - 5 + \frac{2}{3}x = \frac{7}{6}x + 4$. Explain your results.

_____

_____

_____

_____

20. Look for a Pattern Describe the pattern in the equation. Then solve the equation.

$$0.3x + 0.03x + 0.003x + 0.0003x + \ldots = 3$$

_____

_____

_____

_____

21. Critique Reasoning Jared wanted to find three consecutive even integers whose sum was 4 times the first of those integers. He let k represent the first integer, then wrote and solved this equation: $k + (k + 1) + (k + 2) = 4k$. Did he get the correct answer? Explain.

_____

_____

_____

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_____

Equations with the Distributive Property

? ESSENTIAL QUESTION

How do you use the Distributive Property to solve equations?

Using the Distributive Property

The Distributive Property can be useful in solving equations.

EXAMPLE 1   FL 8.EE.3.7b

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A Solve: $3(x - 5) + 1 = 2 + x$

STEP 1 Use the Distributive Property.

$3x - 15 + 1 = 2 + x$ Distribute 3 to the terms within the parentheses.

$3x - 14 = 2 + x$ Simplify.

STEP 2 Use inverse operations to solve the equation.

$$3x - 14 = 2 + x$$
$$\underline{-x \qquad\qquad -x}$$ Subtract x from both sides.
$$2x - 14 = 2$$
$$\underline{+14 \quad +14}$$ Add 14 to both sides.
$$2x = 16$$ Divide both sides by 2.
$$x = 8$$

B Solve: $5 - 7k = -4(k + 1) - 3$

STEP 1 Use the Distributive Property.

$5 - 7k = -4k - 4 - 3$ Distribute -4 to the terms within the parentheses.

$5 - 7k = -4k - 7$ Simplify.

STEP 2 Use inverse operations to solve the equation.

$$5 - 7k = -4k - 7$$
$$\underline{+4k \quad +4k}$$ Add $4k$ to both sides.
$$5 - 3k = -7$$
$$\underline{-5 -5}$$ Subtract 5 from both sides.
$$-3k = -12$$ Divide both sides by -3.
$$k = 4$$

Math Talk
Mathematical Practices

How can you rewrite $7 - (2a + 3) = 12$ without parentheses? Explain your answer.

YOUR TURN

Solve each equation.

1. $y - 5 = 3 - 9(y + 2)$ _____

2. $2(x - 7) - 10 = 12 - 4x$ _____

Using the Distributive Property on Both Sides

Some equations require the use of the Distributive Property on both sides.

EXAMPLE 2 FL 8.EE.3.7b

Solve: $\frac{3}{4}(x - 13) = -2(9 + x)$

Math Talk

Mathematical Practices

How can you eliminate fractions if there is a fraction being distributed on both sides of an equation?

STEP 1 Eliminate the fraction.

$\frac{3}{4}(x - 13) = -2(9 + x)$

$4 \times \frac{3}{4}(x - 13) = 4 \times [-2(9 + x)]$ Multiply both sides by 4.

$3(x - 13) = -8(9 + x)$

STEP 2 Use the Distributive Property.

$3x - 39 = -72 - 8x$ Distribute 3 and −8 to the terms within the parentheses.

STEP 3 Use inverse operations to solve the equation.

$3x - 39 = -72 - 8x$

$\underline{+8x \qquad\qquad +8x}$ Add 8x to both sides.

$11x - 39 = -72$

$\underline{\qquad +39 \quad +39}$ Add 39 to both sides.

$11x = -33$

$\frac{11x}{11} = \frac{-33}{11}$ Divide both sides by 11.

$x = -3$

YOUR TURN

Solve each equation.

3. $-4(-5 - b) = \frac{1}{3}(b + 16)$ _____

4. $\frac{3}{5}(t + 18) = -3(2 - t)$ _____

Solving a Real-World Problem Using the Distributive Property

Solving a real-world problem may involve using the Distributive Property.

EXAMPLE 3 *Problem Solving*   **FL** 8.EE.3.7b

The Coleman family had their bill at a restaurant reduced by $7.50 because of a special discount. They left a tip of $8.90, which was 20% of the reduced amount. How much was their bill before the discount?

Analyze Information

The answer is the amount before the discount.

Formulate a Plan

Use an equation to find the amount before the discount.

Solve

STEP 1   Write the equation $0.2(x - 7.5) = 8.9$, where x is the amount of the Coleman family's bill before the discount.

STEP 2   Use the Distributive Property: $0.2x - 1.5 = 8.9$

STEP 3   Use inverse operations to solve the equation.

$$0.2x - 1.5 = 8.9$$
$$\underline{+ 1.5 \quad + 1.5} \qquad \text{Add 1.5 to both sides.}$$
$$0.2x = 10.4$$
$$\frac{0.2x}{0.2} = \frac{10.4}{0.2} \qquad \text{Divide both sides by 0.2.}$$
$$x = 52$$

The Coleman family's bill before the discount was $52.00.

Justify and Evaluate

$52.00 - $7.50 = $44.50 and 0.2($44.50) = $8.90. This is the amount of the tip the Colemans left. The answer is reasonable.
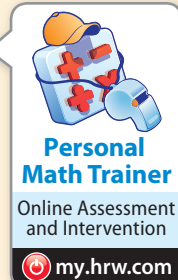
> **Math Talk**
> **Mathematical Practices**
>
> Why do you use 0.2 in Step 1?

YOUR TURN

5. The Smiths spend 8% of their budget on entertainment. Their total budget this year is $2,000 more than last year, and this year they plan to spend $3,840 on entertainment. What was their total budget last year? _____

Guided Practice

Solve each equation.

1. $4(x + 8) - 4 = 34 - 2x$ (Ex. 1)

$$\boxed{}x + \boxed{} - 4 = 34 - 2x$$

$$\boxed{}x + \boxed{} = 34 - 2x$$

$$\boxed{}x + \boxed{} = 34$$

$$\boxed{}x = \boxed{}$$

$$\frac{\boxed{}x}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$$

$$x = \boxed{}$$

2. $\frac{2}{3}(9 + x) = -5(4 - x)$ (Ex. 2)

$$\boxed{} \times \frac{2}{3}(9 + x) = \boxed{} \times [-5(4 - x)]$$

$$\boxed{}(9 + x) = \boxed{}(4 - x)$$

$$\boxed{} + \boxed{}x = \boxed{} \bigcirc \boxed{}x$$

$$\boxed{}x = \boxed{}$$

$$\frac{\boxed{}x}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$$

$$x = \boxed{}$$

3. $-3(x + 4) + 15 = 6 - 4x$ (Ex. 1)

_____

4. $10 + 4x = 5(x - 6) + 33$ (Ex. 1)

_____

5. $x - 9 = 8(2x + 3) - 18$ (Ex. 1)

_____

6. $-6(x - 1) - 7 = -7x + 2$ (Ex. 1)

_____

7. $\frac{1}{10}(x + 11) = -2(8 - x)$ (Ex. 2)

_____

8. $-(4 - x) = \frac{3}{4}(x - 6)$ (Ex. 2)

_____

9. $-8(8 - x) = \frac{4}{5}(x + 10)$ (Ex. 2)

_____

10. $\frac{1}{2}(16 - x) = -12(x + 7)$ (Ex. 2)

_____

11. Sandra saves 12% of her salary for retirement. This year her salary was $3,000 more than in the previous year, and she saved $4,200. What was her salary in the previous year? (Example 3)

Write an equation. _____

Sandra's salary in the previous year was _____.

? ESSENTIAL QUESTION CHECK-IN

12. When solving an equation using the Distributive Property, if the numbers being distributed are fractions, what is your first step? Why?

_____

7.3 Independent Practice

FL **8.EE.3.7b**

13. Multistep Martina is currently 14 years older than her cousin Joey. In 5 years she will be 3 times as old as Joey. Use this information to answer the following questions.

a. If you let x represent Joey's current age, what expression can you use to represent Martina's current age?

_____

b. Based on your answer to part a, what expression represents Joey's age in 5 years? What expression represents Martina's age in 5 years?

_____

c. What equation can you write based on the information given?

_____

d. What is Joey's current age? What is Martina's current age?

_____

14. As part of a school contest, Sarah and Luis are playing a math game. Sarah must pick a number between 1 and 50 and give Luis clues so he can write an equation to find her number. Sarah says, "If I subtract 5 from my number, multiply that quantity by 4, and then add 7 to the result, I get 35." What equation can Luis write based on Sarah's clues and what is Sarah's number?

_____

15. Critical Thinking When solving an equation using the Distributive Property that involves distributing fractions, usually the first step is to multiply by the LCD to eliminate the fractions in order to simplify computation. Is it necessary to do this to solve $\frac{1}{2}(4x + 6) = \frac{1}{3}(9x - 24)$? Why or why not?

_____

_____

_____

16. Solve the equation given in Exercise 15 with and without using the LCD of the fractions. Are your answers the same?

_____

17. **Represent Real-World Problems** A chemist mixed x milliliters of 25% acid solution with some 15% acid solution to produce 100 milliliters of a 19% acid solution. Use this information to fill in the missing information in the table and answer the questions that follow.

	ml of Solution	Percent Acid as a Decimal	ml of Acid
25% Solution	x		
15% Solution			
Mixture (19% Solution)	100		

a. What is the relationship between the milliliters of acid in the 25% solution, the milliliters of acid in the 15% solution, and the milliliters of acid in the mixture? _____

_____

_____

b. What equation can you use to solve for x based on your answer to part a? _____

c. How many milliliters of the 25% solution and the 15% solution did the chemist use in the mixture? _____

_____

H.O.T. **FOCUS ON HIGHER ORDER THINKING**

Work Area

18. **Explain the Error** Anne solved $5(2x) - 3 = 20x + 15$ for x by first distributing 5 on the left side of the equation. She got the answer $x = -3$. However, when she substituted -3 into the original equation for x, she saw that her answer was wrong. What did Anne do wrong, and what is the correct answer? _____

_____

_____

19. **Communicate Mathematical Ideas** Explain a procedure that can be used to solve $5[3(x + 4) - 2(1 - x)] - x - 15 = 14x + 45$. Then solve the equation. _____

_____

_____

_____

Equations with Many Solutions or No Solution

? **ESSENTIAL QUESTION**

How can you give examples of equations with a given number of solutions?

Determining the Number of Solutions

So far, when you solved a linear equation in one variable, you found one value of x that makes the equation a true statement. When you simplify some equations, you may find that they do not have one solution.

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EXAMPLE 1
FL 8.EE.3.7a

Use the properties of equality to simplify each equation. Tell whether the final equation is a true statement.

A $4x - 3 = 2x + 13$

$$4x - 3 = 2x + 13$$

$$\underline{+3 = \quad +3}$$ Add 3 to both sides.

$$4x = 2x + 16$$

$$\underline{-2x \quad -2x}$$ Subtract $2x$ from both sides.

$$2x = 16$$

$$\frac{2x}{2} = \frac{16}{2}$$ Divide both sides by 2.

$$x = 8$$

The statement is true. There is one solution.

B $4x - 5 = 2(2x - 1) - 3$

$$4x - 5 = 2(2x - 1) - 3$$

$$4x - 5 = 4x - 2 - 3$$ Distributive Property

$$4x - 5 = 4x - 5$$ Simplify.

$$\underline{-4x \qquad -4x}$$ Subtract $4x$ from both sides.

$$-5 = -5$$

The statement is true. There are many solutions.

My Notes

C $4x + 2 = 4x - 5$

$$4x + 2 = 4x - 5$$
$$\underline{ - 2 \quad\quad - 2} \quad\quad \text{Subtract 2 from both sides.}$$
$$4x = 4x - 7$$

$$\underline{-4x \quad\quad -4x} \quad\quad \text{Subtract 4x from both sides.}$$
$$0 = -7$$

The statement is false. There is no solution.

Reflect

1. What happens when you substitute any value for x in the original equation in part B? In the original equation in part C?

_____

_____

YOUR TURN

Use the properties of equality to simplify each equation. Tell whether the final equation is a true statement.

2. $2x + 1 = 5x - 8$ **3.** $3(4x + 3) - 2 = 12x + 7$ **4.** $3x - 9 = 5 + 3x$

_____   _____   _____

Writing Equations with a Given Number of Solutions

When you simplify an equation using the properties of equality, you will find one of three results.

Result	What does this mean?	How many solutions?
$x = a$	When the value of x is a, the equation is a true statement.	1
$a = a$	Any value of x makes the equation a true statement.	Infinitely many
$a = b$, where $a \neq b$	There is no value of x that makes the equation a true statement.	0

You can use these results to write a linear equation that has a given number of solutions.
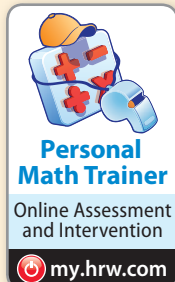
EXAMPLE 2 FL 8.EE.3.7a

Write a linear equation in one variable that has no solution.

You can use the strategy of working backward:

STEP 1 Start with a false statement such as $3 = 5$. Add the same variable term to both sides.

$$3 + x = 5 + x \qquad \text{Add } x \text{ to both sides.}$$

STEP 2 Next, add the same constant to both sides and combine like terms on each side of the equation.

$$10 + x = 12 + x \qquad \text{Add 7 to both sides.}$$

STEP 3 Verify that your equation has no solutions by using properties of equality to simplify your equation.

$$10 + x = 12 + x$$
$$\underline{\quad - x = \quad - x}$$
$$10 = 12$$

Reflect

5. Explain why the result of the process above is an equation with no solution.

_____

_____

_____

_____

YOUR TURN

Tell whether each equation has one, zero, or infinitely many solutions.

6. $6 + 3x = x - 8$ _____

7. $8x + 4 = 4(2x + 1)$ _____

Complete each equation so that it has the indicated number of solutions.

8. No solution: $3x + 1 = 3x +$ _____

9. Infinitely many: $2x - 4 = 2x -$ _____

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Guided Practice

Use the properties of equality to simplify each equation. Tell whether the final equation is a true statement. (Example 1)

1. $3x - 2 = 25 - 6x$

$\underline{+\,6x} \qquad\quad \underline{+\,6x}$

$\boxed{} - 2 = \boxed{}$

$\boxed{} = \boxed{}$

$\overline{\boxed{}\,x = \boxed{}}$

$\dfrac{\boxed{}\,x}{\boxed{}} = \dfrac{\boxed{}}{\boxed{}}$

$x = \boxed{}$

The statement is $\boxed{}$.

2. $2x - 4 = 2(x - 1) + 3$

$2x - 4 = \boxed{} + 3$

$2x - 4 = 2x + \boxed{}$

$\dfrac{-\boxed{}}{} = \dfrac{-\boxed{}}{}$

$\boxed{} = \boxed{}$

The statement is $\boxed{}$.

3. How many solutions are there to the equation in Exercise 2? (Example 1) _____

4. After simplifying an equation, Juana gets $6 = 6$. Explain what this means. (Example 1)

_____

Write a linear equation in one variable that has infinitely many solutions. (Example 2)

5. Start with a _____ statement.

Add the _____ to both sides.

Add the _____ to both sides.

Combine _____ terms.

$10 = \boxed{}$

$10 + x = \boxed{}$

$10 + x + 5 = \boxed{}$

$\boxed{} = \boxed{}$

? **ESSENTIAL QUESTION CHECK-IN**

6. Give an example of an equation with an infinite number of solutions. Then make one change to the equation so that it has no solution.

_____

_____

7.4 Independent Practice

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Tell whether each equation has one, zero, or infinitely many solutions.

7. $-(2x + 2) - 1 = -x - (x + 3)$

8. $-2(z + 3) - z = -z - 4(z + 2)$

_____

_____

Create an equation with the indicated number of solutions.

9. No solution:

$3\left(x - \dfrac{4}{3}\right) = 3x + \boxed{}$

10. Infinitely many solutions:

$2(x - 1) + 6x = 4\left(\boxed{} - 1\right) + 2$

11. One solution of $x = -1$:

$5x - (x - 2) = 2x - \left(\boxed{}\right)$

12. Infinitely many solutions:

$-(x - 8) + 4x = 2\left(\boxed{}\right) + x$

13. Persevere in Problem Solving The Dig It Project is designing two gardens that have the same perimeter. One garden is a trapezoid whose nonparallel sides are equal. The other is a quadrilateral. Two possible designs are shown at the right.

$2x - 2$

$x + 1$ $x + 1$

x
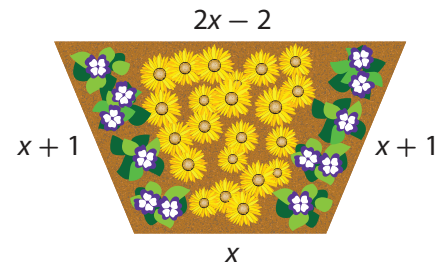
a. Based on these designs, is there more than one value for x? Explain how you know this.

_____

_____

_____

_____

_____

$2x - 9$

x $x + 1$

$x + 8$
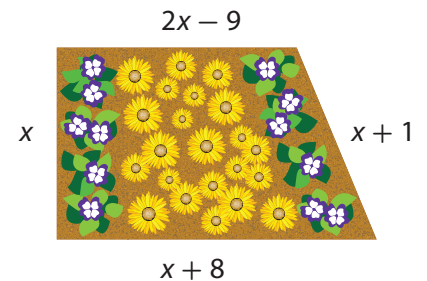
**b.** Why does your answer to part a make sense in this context?

_____

_____

_____

c. Suppose the Dig It Project wants the perimeter of each garden to be 60 meters. What is the value of x in this case? How did you find this?

_____

_____

14. Critique Reasoning Lisa says that the indicated angles cannot have the same measure. Marita disagrees and says she can prove that they can have the same measure. Who do you agree with? Justify your answer.

$(9x - 25 + x)°$

$(x + 50 + 2x - 12)°$

_____

_____

_____

_____

_____
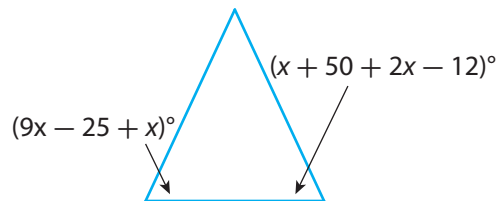
15. Represent Real-World Problems Adele opens an account with $100 and deposits $35 a month. Kent opens an account with $50 and also deposits $35 a month. Will they have the same amount in their accounts at any point? If so, in how many months and how much will be in each account? Explain.

_____

_____

_____

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

16. Communicate Mathematical Ideas Frank solved an equation and got the result $x = x$. Sarah solved the same equation and got $12 = 12$. Frank says that one of them is incorrect because you cannot get different results for the same equation. What would you say to Frank? If both results are indeed correct, explain how this happened.

_____

_____

_____

17. Critique Reasoning Matt said $2x - 7 = 2(x - 7)$ has infinitely many solutions. Is he correct? Justify Matt's answer or show how he is incorrect.

_____

_____

_____

Ready to Go On?

7.1 Equations with the Variable on Both Sides

Solve.

1. $4a - 4 = 8 + a$ _____

2. $4x + 5 = x + 8$ _____

3. Hue is arranging chairs. She can form 6 rows of a given length with 3 chairs left over, or 8 rows of that same length if she gets 11 more chairs. Write and solve an equation to find how many chairs are in that row length.

_____

7.2 Equations with Rational Numbers

Solve.

4. $\frac{2}{3}n - \frac{2}{3} = \frac{n}{6} + \frac{4}{3}$ _____

5. $1.5d + 3.25 = 1 + 2.25d$ _____

6. Happy Paws charges $19.00 plus $1.50 per hour to keep a dog during the day. Woof Watchers charges $15.00 plus $2.75 per hour. Write and solve an equation to find for how many hours the total cost of the services is equal.

_____

7.3 Equations with the Distributive Property

Solve.

7. $14 + 5x = 3(-x + 3) - 11$ _____

8. $\frac{1}{4}(x - 7) = 1 + 3x$ _____

9. $-5(2x - 9) = 2(x - 8) - 11$ _____

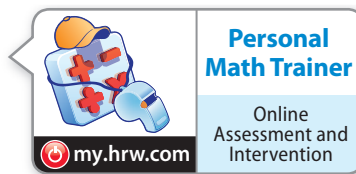**10.** $3(x + 5) = 2(3x + 12)$ _____

7.4 Equations with Many Solutions or No Solution

Tell whether each equation has one, zero, or infinitely many solutions.

11. $5(x - 3) + 6 = 5x - 9$ _____

12. $5(x - 3) + 6 = 5x - 10$ _____

13. $5(x - 3) + 6 = 4x + 3$ _____

Selected Response

1. Two cars are traveling in the same direction. The first car is going 40 mi/h, and the second car is going 55 mi/h. The first car left 3 hours before the second car. Which equation could you solve to find how many hours it will take for the second car to catch up to the first car?

Ⓐ $55t + 3 = 40t$

Ⓑ $55t + 165 = 40t$

Ⓒ $40t + 3 = 55t$

Ⓓ $40t + 120 = 55t$

2. Which linear equation is represented by the table?

x	−2	1	3	6
y	7	4	2	−1

Ⓐ $y = -x + 5$ Ⓒ $y = x + 3$

Ⓑ $y = 2x - 1$ Ⓓ $y = -3x + 11$

3. Shawn's Rentals charges $27.50 per hour to rent a surfboard and a wetsuit. Darla's Surf Shop charges $23.25 per hour to rent a surfboard plus $17 extra for a wetsuit. For what total number of hours are the charges for Shawn's Rentals the same as the charges for Darla's Surf Shop?

Ⓐ 3 Ⓒ 5

Ⓑ 4 Ⓓ 6

4. Which of the following is irrational?

Ⓐ −8 Ⓒ $\sqrt{11}$

Ⓑ 4.63 Ⓓ $\frac{1}{3}$

5. Greg and Jane left a 15% tip after dinner. The amount of the tip was $9. Greg's dinner cost $24. Which equation can you use to find x, the cost of Jane's dinner?

Ⓐ $0.15x + 24 = 9$

Ⓑ $0.15(x + 24) = 9$

Ⓒ $15(x + 24) = 9$

Ⓓ $0.15x = 24 + 9$

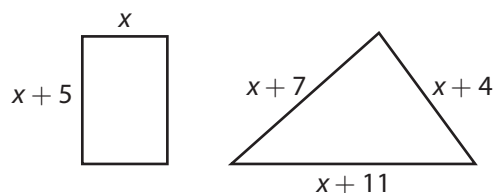**6.** For the equation $3(2x - 5) = 6x + k$, which value of k will create an equation with infinitely many solutions?

Ⓐ 15 Ⓒ 5

Ⓑ −5 Ⓓ −15

7. Which of the following is equivalent to 2^{-4}?

Ⓐ $\frac{1}{16}$ Ⓒ −2

Ⓑ $\frac{1}{8}$ Ⓓ −16

Mini-Task

8. Use the figures below for parts a and b.

a. Both figures have the same perimeter. Solve for x.

_____

b. What is the perimeter of each figure?

_____